Assignment 11

Coverage: 16.6 (cont'd), 16.7

Exercises: 16.6 no. 21, 24, 27; 16.7 no. 3, 6, 4, 8, 13, 18.

Hand in 16.6 no 21, 24 ; 16.7 no 8, 13 and Supplementary Problem no 1 by Nov 29.

Supplementary Problems

1. Let S be the triangle with vertices at $(1,0,0),(0,2,0),(0,0,7)$ with normal pointing upward. Verify Stokes' theorem for the vector field $\mathbf{F} = x\mathbf{i} + 3z\mathbf{j}$.

2. Show that for a closed oriented surface S, that is, a surface without boundary,

$$\iint_S \nabla \times \mathbf{F} \cdot \mathbf{n} \, d\sigma = 0 \ .$$

   Hint: See how to apply Stokes' theorem.

3. (Optional) Let S be the surface given by $(x,y) \mapsto (x,y,f(x,y)), (x,y) \in D$. That is, it is the graph of f over the region D. Show that in this case Stokes' theorem

$$\iint_S \nabla \times \mathbf{F} \, d\sigma = \oint_C \mathbf{F} \cdot d\mathbf{r} \ ,$$

   ($\mathbf{F}$ is a smooth vector field on S) can be deduced from Green's theorem for some vector field on D. Hint: Let the boundary of D be $\mathbf{r}(t) = (x(t), y(t))$. Then the boundary of S is $\mathbf{c}(t) = (x(t), y(t), f(x(t), y(t)))$. Convert the integration in S and C to the integration on D and the boundary of D respectively.